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DESCRIPTION OF ROTATIONAL BANDS AT LOW AND HIGH SPINS IN RARE EARTH AND ACTINIDE DEFORMED NUCLEI USING THE THREE PARAMETRIC EXPRESSION

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ABSTRACT

The variable moment of inertia with softness VMIS is considered to analyze the yarst state rotational bands of even-even deformed nuclei in the rare-earth and actinide region including high spin state. This model gives good agreement with the experimental data and fairly accurate description of the backbending phenomena in the rare-earth and actinide regions.

KEYWORDS: Rotational Bands, High Spins in Rare Earth, Actinide Deformed Nuclei using, Parametric Expression

INTRODUCTION p

In recent years, several semi-empirical semi-classical models have been introduced for correlating the large amount of experimental data available for the energy levels of the ground state bands in even-even nuclei. In particular, the variable moment of inertia(VMI)model [1,2]and its phenomenological equivalent, the cranking model [3], have been accepted as giving very good descriptions of ground state bands, and also $\beta$and γ bands, of even-even nuclei up to the point where the backbending occurs.

Harris model [3, 4], Sood model [5] and the two parameters expression [6,7] are different models used to get a good agreement with the experimental results.

In the present work we use the three parametric expression (the variable moment of inertia with softness VMIS) [8],and the two parameter model to make a comparison with the experimental data., we have able to study different phenomena in all known cases in medium and heavy even-even nuclei, (more than 140 nuclei) i.e) in the Pd region, the Ba-Ce region, the rare earth region, and Pu244which lie in the actinide region. In this analysis we calculate the value of R4, the softness S, and study the backbending phenomena.

The rotational spectra is presented for Pu^{240},Th^{230}, Os^{192},Os^{170}, Er^{162}, Dy^{160}, Nd^{150}, Xe^{122},$Pd^{110}$and Zr^{102} up to I~20^{+}.In the next section the proposed formula is given. In section 3 the results are presented and discussed.

Formation

According to the Bohr-Mottelson model[9] the lowest rotational energy levels for even-even nuclei are given by the formula

$$E(I) = AI(I+1) = \frac{I(I+1)}{2\Im} \tag{1}$$

$A = \frac{1}{2\Im}$, where $\Im$ is the nuclear moment of inertia, and I is the angular momentum.

With some approximation of Bohr Hamiltonian [10] have derived a simple two parameters formula for collective spectra of well deformed nuclei with a simple axial symmetry, which was obtained by Holmberg and Lipas [11].the moment of inertia increases approximately linearly with rotational energy levels i.e.

$$\Im = C_1 + C_2 E \tag{2}$$

Where C_1, C_2 are positive constants. From equations (1), (2) one obtains

$$E = a \left\{\sqrt{1 + bI(I+1)} - 1\right\} \tag{3}$$

Where $2a = \frac{C_1}{C_2}$, $b = \frac{2}{aC_1}$

The ground state rotational bands of even-even nuclei in the rare-earth and actinide region for spins I ~20 shows an anomalous behavior in the nuclear rotational motion for large values of angular momentum. Usually the rotational frequency $\hbar\omega$ and the moment of inertia $\Im$ are deduced from the transition energy by defining

$$\hbar\omega = \frac{dE(I)}{d\sqrt{I(I+1)}} \tag{4}$$

$$\frac{2\Im}{\hbar^2} = \left(\frac{dE(I)}{dI(I+1)}\right)^{-1} \tag{5}$$

We then employ equation (4) and (5) to deduce the most sensitive relation expressive of $\Im$ and ω^2, respectively given

$$\frac{2\Im}{\hbar^2} = \frac{4I-2}{\Delta E_\gamma}$$

$$(\hbar\omega)^2 = (I^2 + I + 1)\left[\frac{\Delta E_\gamma}{2I-1}\right]^2$$

Where

$\Delta E_\gamma = E(I) - E(I-2)$ is the observed energy difference between neighboring levels.

In present work we use the three parametric expression, the variable moment of inertia with softness VMIS [8].We have calculated the energy levels in ground state rotational band of deformed nuclei using the form

$$E(I) = \frac{A_0}{1+\sigma I}\; I(I+1) - C{A_0}^3 \frac{1-2\sigma I}{1+\sigma I} I^2 (I+1)^2 \tag{6}$$

Where $A_0 = \frac{\hbar^2}{2\Im}$

And the softness parameter σ is given by [12]

$$\sigma_n = \frac{1}{n!}\frac{1}{\Im}\frac{\partial^n \Im(I)}{\partial I^n}\bigg|_{I=0} \tag{7}$$

Where $\Im$ being the unperturbed nuclear moment of inertia[13],and the constant C is connected with β - and γ -vibrational energies through the relation [14]

$$C = \frac{12}{(\hbar\omega_\beta)^2} + \frac{4}{(\hbar\omega_\gamma)^2} \tag{8}$$

Here $\hbar\omega_\beta$ and $\hbar\omega_\gamma$are the head energies of these vibrations, respectively.

Calculations and Results

In this paper we take $\mathbf{A_0}$**,** σ and **C** as three free parameters of the VMIS model, which are adjusted by minimizing equation (6) to give a least square fit to experiments for low and high angular momentum.The calculations used the two parameter expression, equation (3), are carried out in similar manner as for the VMIS model. The parameters $\mathbf{A_0}$

σ, **C**,**a** and **b**are listed in table 1.

The results of our calculations for the ground state band up to spin I =20 for the chosen nuclei are presented in table (2)[are also shown in figure (1-9)]. In this table, the first, second and fourth row contains the experimental energy [15, 16], the VMIS energy and the two parameter model energy. The six and seven row represent the rotational constant for experimental and VMIS data. The value of the parameter A is determine from the experimental value E(4) or E(2).

By comparing between the two parameters formula model and the VMIS model, at low spins a very successful description is the two parameter VMI model, equation (3), which give a simple linear dependence of $\Im$ and $\hbar^2\omega^2$, and are in good agreement with the experimental data. For large values of spins the results calculated by the two parameters VMI are not in agreement with the experimental data for the rare earth nuclei but give good agreement for the actinide nuclei .Table 2 and figures (1-9) shows a good agreement between the experimental energy Eexp and the energy calculated by using VMIS model. Also we shows a good agreement between Aexp and AVMIS.

The softness S of these nuclei indicates that the rotational nuclei are very close to each other. The rotational nuclei are divided into two groups, stretching nuclei, with positive S value, as soft rotors and shrinking nuclei, with negative S value, as hard rotors, table 3.

The nuclei in our investigation can be divided into three groups, table (4) ,according to the value of R4, where

Region (I): $2 \le R4 \le 2.4$, for vibration nuclei,(e.g.) Ba140…

Region (II): 2.4 ≤ R4 ≤ 3, for transitional nuclei,(e.g.) Zr100, Pd110, Xe122,…

Region (III): 3 ≤ R4 ≤ 10/3, for rotational nuclei,(e.g.) Dy160, Er162, Yb164, Gd158,…

For our calculations of the softness S and R4 it is clear that most of the nuclei under investigation (e.g. more than 30 nuclei) lie in the rotational region.

From figures (10-15), a systematic study of the level structure up to spin 24^+ of six selected nuclei including soft as well as good rotators and exhibit back bending or upbending are performed. The absence of backbending in some nuclei may be ascribed to the presence of stable octupole deformation in them.

We conclude that the VMIS model is a successful tool in studding groundstate energy levels in deformed nuclei up to high spin states. Good agreement was noticed between the calculations by using VMIS model and the experimental data.

CONCLUSIONS

We conclude that the VMIS model is a successful tool in studding ground state energy levels in deformed nuclei up to high spin states. Good agreement was noticed between the calculations by using VMIS model and the experimental data.

Table 1: Fitted Parameters of Equations (3) and (6)

Nuclei	VMIS Equ.(6)			Two Parameter Model Equ.(3)	
	A_0	σ	C	a	b
Zr^{102}	0.02557	0.70338	0.01548	11.0592	0.00429
Pd^{110}	0.08781	0.01796	0.22669	0.58698	0.27984
Xe^{122}	0.065	0.02645	0.14129	0.581338	0.2113986
Ba^{140}	0.5374	0.00962	2.20139	0.039247	5.153
Nd^{150}	0.02414	0.23934	0.06695	0.61084	0.008193
Dy^{160}	0.0147	3.38056	0.00467	5.3404	0.005455
Er^{162}	0.01706	3.45923	0.00526	4.2645211	0.00893873
Os^{170}	0.06002	-0.05931	0.19396	0.30308	0.46444
Os^{192}	0.03769	0.18735	0.07411	0.93124	0.081765
Th^{230}	0.00908	5.25314	0.00926	3.30316	0.00541
Pu^{240}	0.00741	2.50813	0.00766	6.514	0.0021974

Table 2: Experimental and Calculated Energy Levels in [MeV] of the Ground State Rotational Band for Selected Nuclei under studied. The Six and Seven Row Give the Experimental and VMIS Rotational Constant

$$\left[\ x_1 = \left(\frac{E_{exp} - E_{vmis}}{E_{exp}}\right) \mathrm{x}100,\ x_2 = \left(\frac{E_{exp} - E_{TPM}}{E_{exp}}\right) \mathrm{x}100\ \right]$$

Nuclei		2+	4+	6+	8+	10+	12+	14+	16+	18+	20+	22+	24+
Zr^{102}	E_{exp}	0.15178	0.47828	0.96478	1.5949	2.3515	3.2123						
	E_{VMIS}	0.14844	0.47775	0.96732	1.59754	2.35086	3.2127						
	X_1	2.2	.11	-0.263	-0.165	0.029	-0.018						
	E_{TPM}	0.14142	0.46467	0.95508	1.5932	2.358	3.229						
	X_2	6.82	2.84	1	0.1	-0.276	-0.519						
	A_{exp}	0.0234	0.02332	0.02211	0.021	0.0199	0.01871						
	A_{VMIS}	0.0236	0.02352	0.02225	0.021	0.01982	0.018735						
Pd^{110}	E_{exp}	0.3738	0.9207	1.574	2.296	3.131	4.03						
	E_{VMIS}	0.3624	0.923	1.5781	2.3058	3.1156	4.035						
	X_1	3.049	-0.249	-0.26	-0.426	0.491	-0.123						
	E_{TPM}	0.37379	0.9206	1.50922	2.1123	2.7221	3.3355						
	X_2	0.0026	0.01	4.11	8.00	13.06	17.23						
	A_{exp}	o.04	0.03906	0.02969	0.02406	0.02197	0.01954						
	A_{VMIS}	0.04	0.04004	0.02977	0.02425	0.02131	0.01998						
Xe^{122}	E_{exp}	0.3311	0.8283	1.466	2.217	3.039	3.919						
	E_{VMIS}	0.304	0.830	1.482	2.219	3.029	3.918						
	X_1	8.18	-0.205	-1.09	-0.09	0.329	0.0255						
	E_{TPM}	0.294	0.747	1.245	1.76	2.281	2.807						
	X_2	11.2	70.17	15.07	20.61	24.94	28.37						
	A_{exp}	0.037	0.033	0.027	0.0242	0.021	0.0187						
	A_{VMIS}	0.065	0.035	0.028	0.0237	0.0207	0.0189						

Table 2: Continue

Nuclei		2+	4+	6+	8+	10+	12+	14+	16+	18+	20+	22+	24+
Nd^{150}	E_{exp}	0.1352	0.3814	0.7204	1.1297	1.599	2.119	2.682					
	E_{VMIS}	0.1276	0.3803	0.7225	1.1326	1.5988	2.1157	2.6838					
	X_1	5.621	0.288	-0.291	-0.256	0.0125	0.155	-0.067					
	E_{TPM}	0.13518	0.38139	0.6764	0.9935	1.322	1.6567	1.9955					
	X_2	0.0147	0.00262	6.107	11.378	17.323	21.816	25.59					
	A_{exp}	0.018	0.01758	0.0154	0.01364	0.0123	0.0113	0.0104					
	A_{VMIS}	0.019	0.018	0.0155	0.0136	0.0122	0.0112	0.01092					
Dy^{160}	E_{exp}	0.0867	0.2838	0.5811	0.9668	1.4287	1.9515	2.515	3.0917	3.6722			
	E_{VMIS}	0.0869	0.2844	0.583	0.9704	1.432	1.9515	2.5108	3. 091	3.6734			
	X_1	-0.23	-0.211	-0.326	-0.165	-0.23	0.0	0.167	0.022	-0.032			
	E_{TPM}	0.08669	0.28377	0.58025	0.9608	1.4148	1.9252	2.482	3.076	3.6998			
	X_2	0.0115	0.0105	0.1428	0.6206	0.9729	1.3476	1.3121	0.5078	-.7515			
	A_{exp}	0.015	0.014	0.0135	0.0128	0.0121	0.0113	0.0104	0.0093	0.0082			
	A_{VMIS}	0.015	0.014	0.0135	0.0129	0.0121	0.0112	0.0103	0.0093	0.0083			
Er^{162}	E_{exp}	0.102	0.329	0.666	1.096	1.602	2.165	2.745	3.229	3.846	4.462		
	E_{VMIS}	0.100	0.329	0.667	1.100	1.606	2.160	2.735	3.305	3.842	4.318		
	X_1	1.96	0.0	-0.15	-0.365	-0.249	0.23	0.364	-2.35	0.104	3.22		
	E_{TPM}	0.112	0.365	0.736	1.202	1.741	2.33	2.943	3.582	4.241	5.033		
	X_2	-9.8	-10.94	-10.51	-9.67	-9.05	-7.62	-7.213	-10.93	6.86	-12.79		
	A_{exp}	0.0165	0.01625	0.0153	0.0143	0.0133	0.0122	0.0107	0.0088	0.0079	0.0079		
	A_{VMIS}	0.0164	0.0162	0.0154	0.0014	0.0131	0.012	0.0106	0.0091	0.0076	0.0061		

Table 2: Continue

Nuclei		2+	4+	6+	8+	10+	12+	14+	16+	18+	20+	22+	24+
Os^{170}	E_{exp}	0.2867	0.7499	1.3254	1.9458	2.5452							
	E_{VMIS}	0.2782	0.7545	1.3284	1.9415	2.5463							
	X_1	2.964	-0.613	-0.226	0.22	-0.043							
	E_{TPM}	0.28669	0.6690	1.0693	1.4755	1.8843							
	X_2	0.00348	10.788	19.322	24.17	25.966							
	A_{exp}	0.034	0.033	0.0261	0.0206	0.0157							
	A_{VMIS}	0.035	0.034	0.026	0.0204	0.0159							
Os^{192}	E_{exp}	0.2057	0.5802	1.0892	1.7083	2.4188	3.211						
	E_{VMIS}	0.19761	0.5801	1.0943	1.7095	2.4146	3.2123						
	X_1	3.932	0.017	-0.468	-0.07	0.173	0.04						
	E_{TPM}	0.2057	0.5805	1.0297	1.51263	2.0127	2.5225						
	X_2	0.0	-0.0827	5.462	11.454	16.798	21.441						
	A_{exp}	0.029	0.0267	0.0231	0.0206	0.0186	0.0172						
	A_{VMIS}	0.0291	0.0273	0.0233	0.0205	0.0185	0.0173						
Th^{230}	E_{exp}	0.0532	0.1741	0.3566	0.5941	0.8797	1.2078	1.5729	1.9715	2.3978	2.85	3.325	3.812
	E_{VMIS}	0.0533	0.1737	0.3554	0.5925	0.8787	1.2079	1.574	1.972	2.399	2.849	3.321	3.814
	X_1	0.187	0.229	0.336	0.269	0.113	-0.82	-0.07	-0.025	-0.05	-.035	0.12	-.052
	E_{TPM}	0.05318	0.1741	0.35608	0.59053	0.86864	1.1822	1.5245	1.8897	2.2734	2.672	3.0826	3.5032
	X_2	0.0375	0.0	0.1458	0.601	1.2572	2.1195	3.0771	4.1491	5.188	6.2456	7.2902	8.1
	A_{exp}	0.0089	0.00863	0.00829	0.00791	0.00751	0.00713	0.0067	0.0064	0.006	0.0057	0.0055	0.00518
	A_{VMIS}	0.0088	0.0086	0.00825	0.0079	0.0075	0.00715	0.0067	0.0064	0.0061	0.00576	0.00548	0.00524
Pu^{240}	E_{exp}	0.0428	0.1416	0.2943	0.4975	0.7478	1.0418	1.3756	1.7456	2.152	2.591	3.56	4.088
	E_{VMIS}	0.0437	0.1434	0.2961	0.4985	0.7477	1.0405	1.374	1.7454	2.152	2.5915	3.5607	4.087
	X_1	-2.102	-1.27	-0.611	-0.201	0.013	0.124	0.116	-2.28	0.0	-.019	-1.705	0.0244
	E_{TPM}	0.0428	0.1416	0.29396	0.4964	0.7447	1.0343	1.3608	1.7197	2.107	2.5189	2.9523	3.4045
	X_2	0.0	0.0	0.1155	0.2211	0.4145	0.7199	1.0977	1.4837	2.091	2.7827	17.07	16.719
	A_{exp}	0.008	0.00705	0.00694	0.00677	0.00658	0.00639	0.00618	0.00596	0.0058	0.00562	0.00546	0.0053
	A_{VMIS}	0.0083	0.00712	0.00694	0.00674	0.00655	0.00636	0.00617	0.00599	0.0058	0.00563	0.00546	0.0053

Table 3: Distribution of Nuclei According to Softness S.

Stretching Nuclei (+ve S)	Shrinking Nuclei (-ve S)
Dy^{158} Yb^{164} Er^{160} Ce^{132}	$Dy^{160-162-164-}$ Yb^{168} $Er^{162-164-166}$ Ce^{134} $Hf^{172-174-176}$ W^{182} Sm^{154} Gd^{158}

Table 4: Distribution of Nuclei According to R_4

$2 \leq R_4 \leq 2.4$, for Vibration Nuclei	$2.4 \leq R_4 \leq 3$, for Transitional Nuclei	$3 \leq R_4 \leq 10/3$, for Rotational Nuclei	
Ba^{140}	Zr^{100},	Dy^{160}	Yb^{168}
Os^{180}	Pd^{110}	Er^{162}	Yb^{170}
	Xe^{122}	Yb^{164}	Hf^{172}
	Pd^{110}	Gd^{158}	Hf^{174}
	Ba^{120}	Sm^{154}	Hf^{176}
	Xe^{122}	Gd^{158}	Os^{182}
	Nd^{132}	Dy^{158}	W^{182}
	Ce^{132}	Dy^{160}	U^{232}
	Ce^{134}	Dy^{162}	U^{234}
	Er^{158}	Dy^{164}	U^{236}
	Os^{192}	Er^{160}	U^{238}
	Os^{190}	Er^{162}	Th^{230}
	Os^{172}	Er^{164}	Th^{232}
	O^{170}	Er^{166}	Pu^{240}
		Yb^{164}	Pu^{242}
		Yb^{166}	

Figure captions

Figure (1-9): Shows the relation between rotational energy E and angular momentum I for the eight nuclei.

Figure (10-15): Shows the relation between $\frac{2\Im}{\hbar^2}$ and $(\hbar\omega)^2$.

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APPENDICES

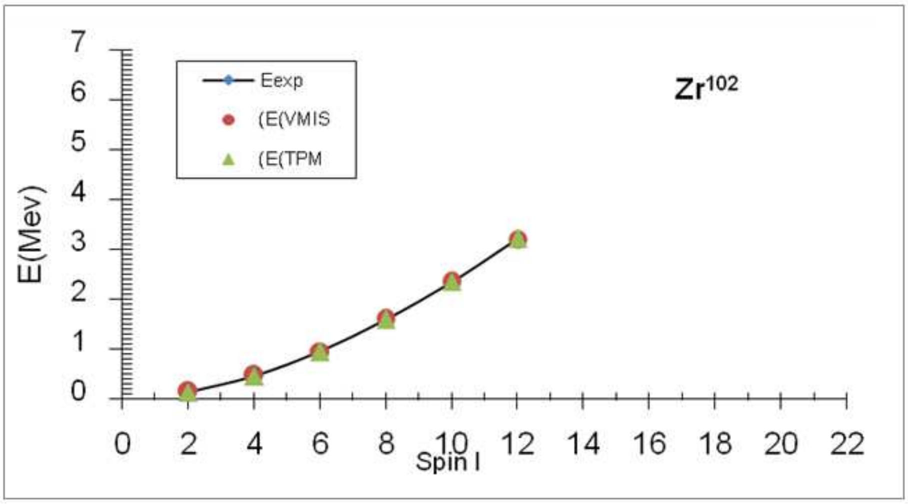


Figure 1

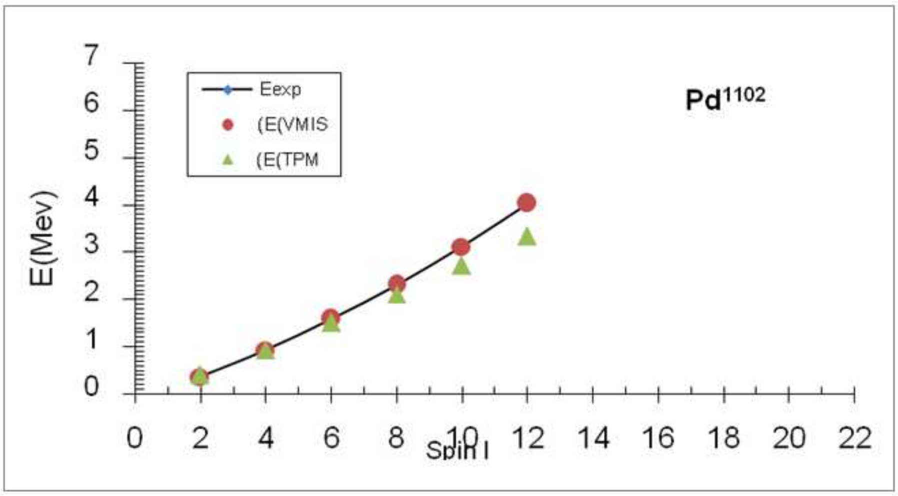
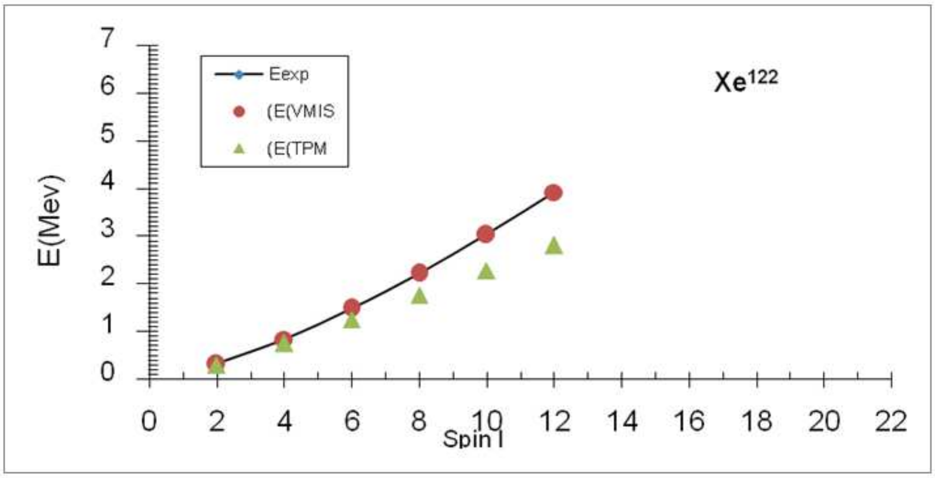


Figure 2



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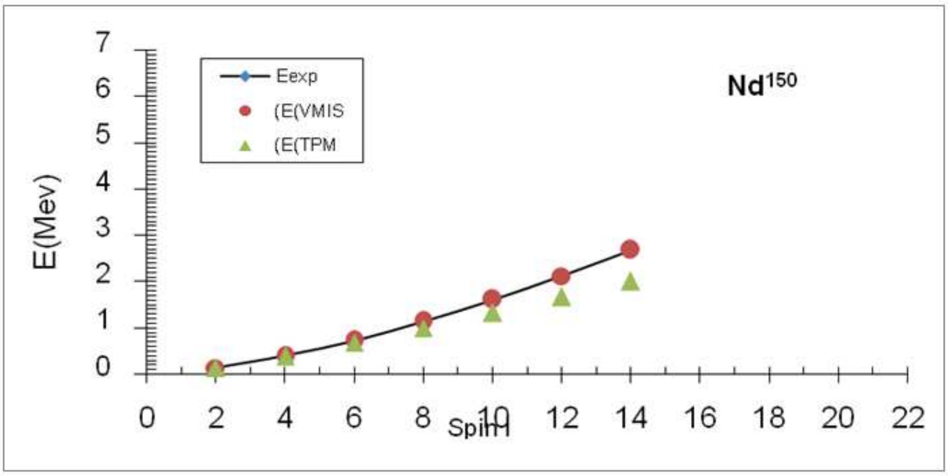
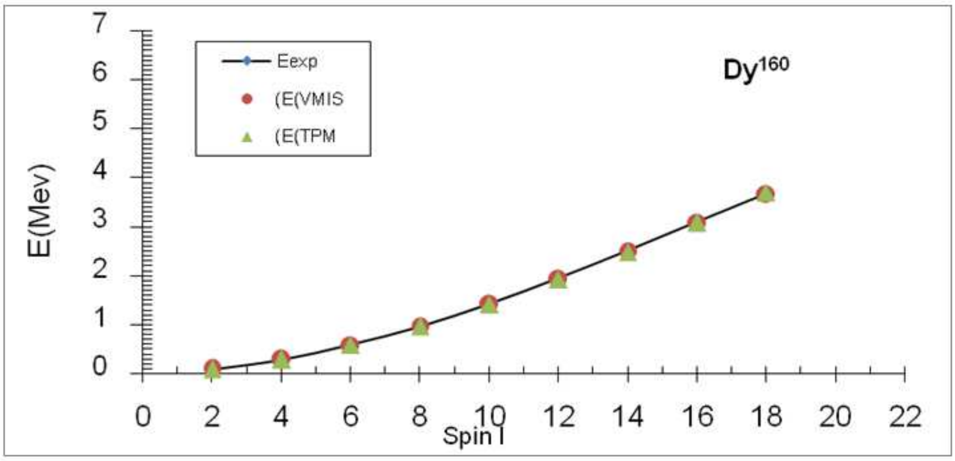


Figure 4



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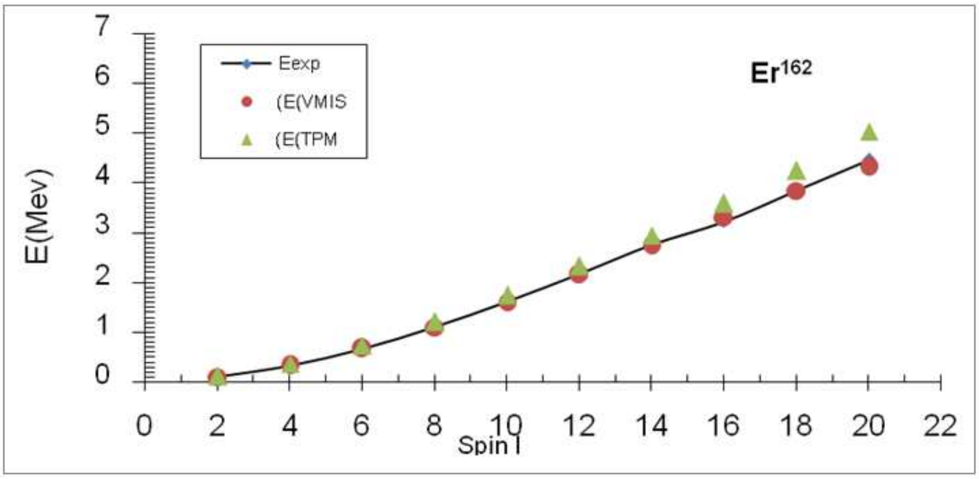

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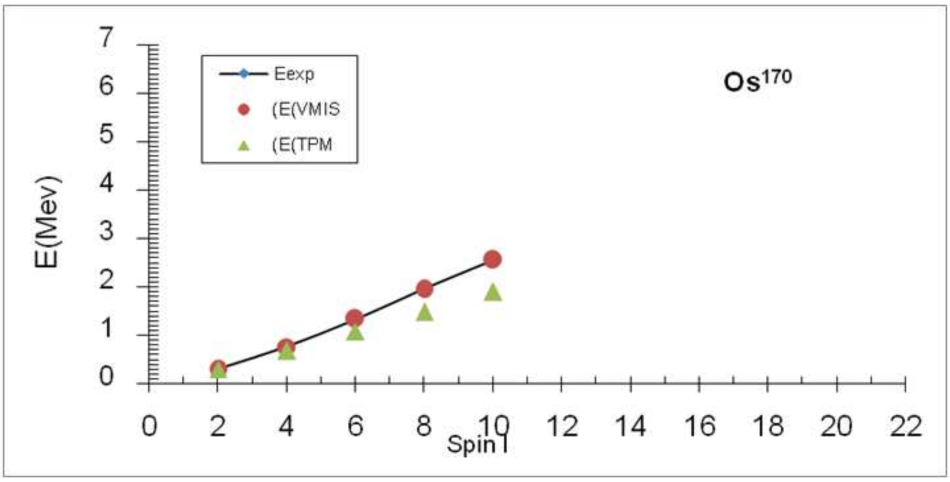

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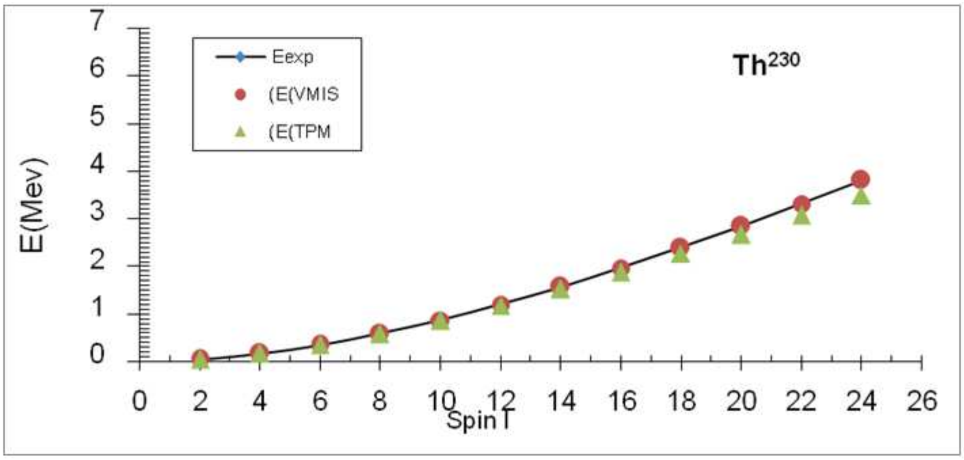

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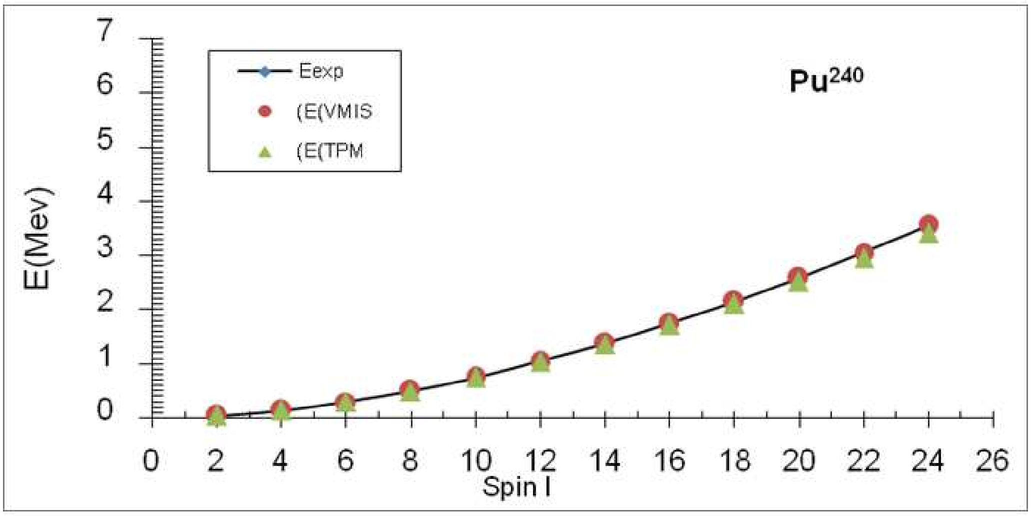

Figure 9

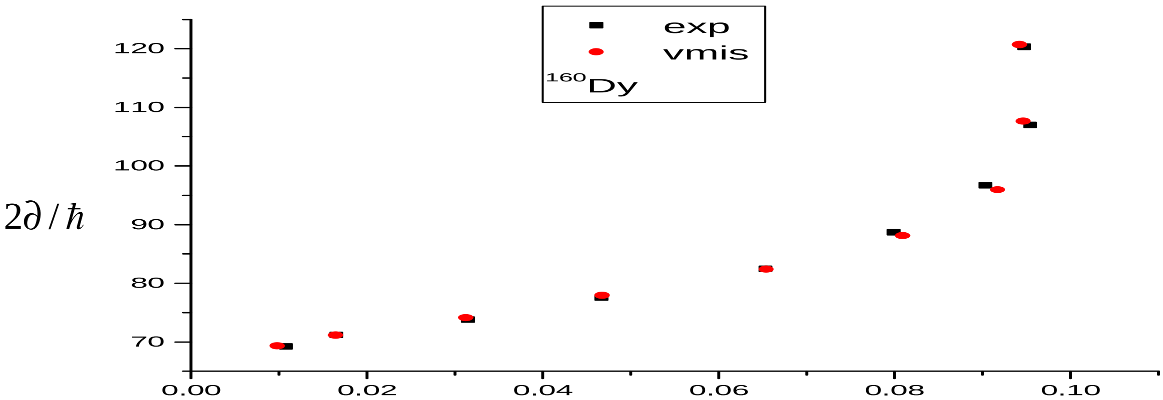

Figure 10: $(\hbar\omega)^2$

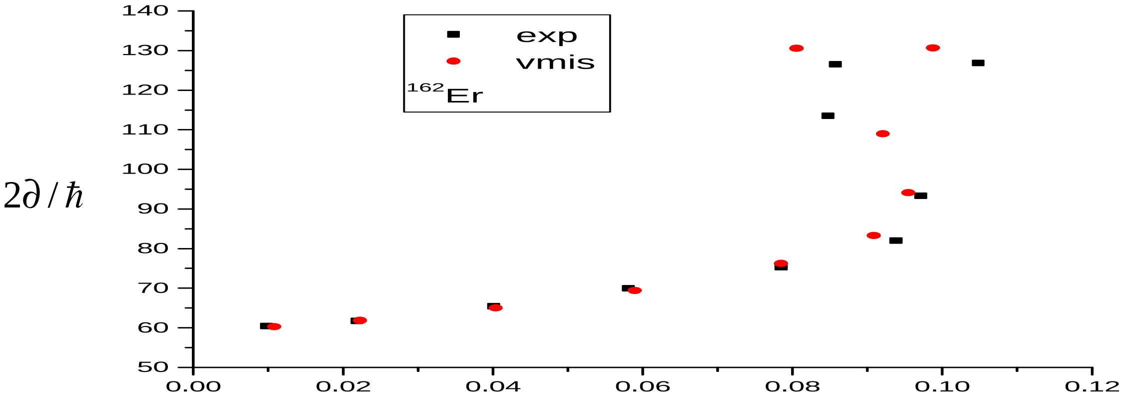

Figure 11: $(\hbar\omega)^2$

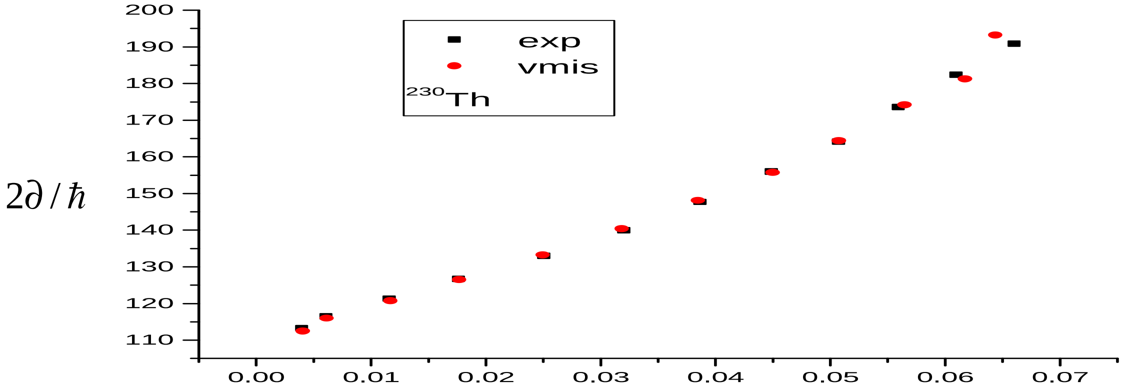

Figure 12: $(\hbar\omega)^2$

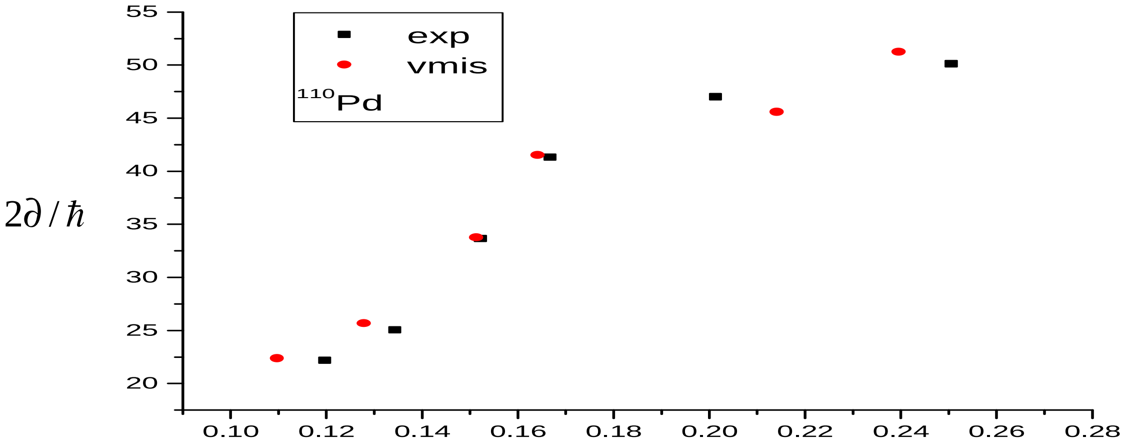

Figure 13: $(\hbar\omega)^2$

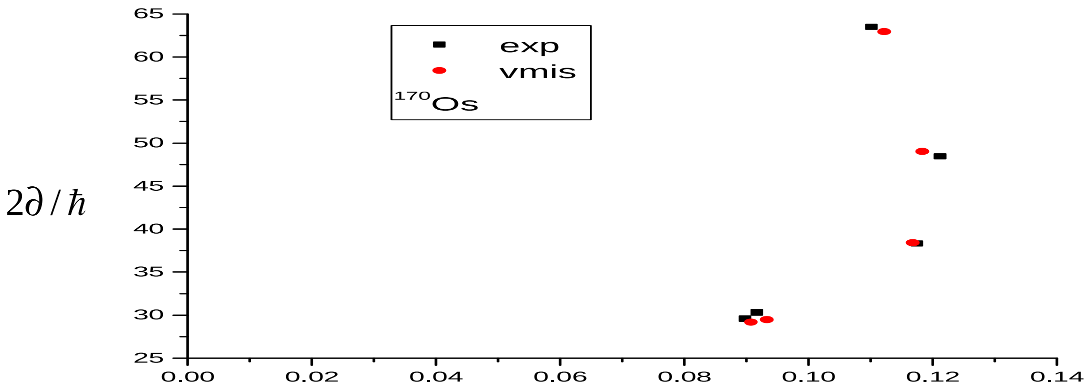

Figure 14: $(\hbar\omega)^2$

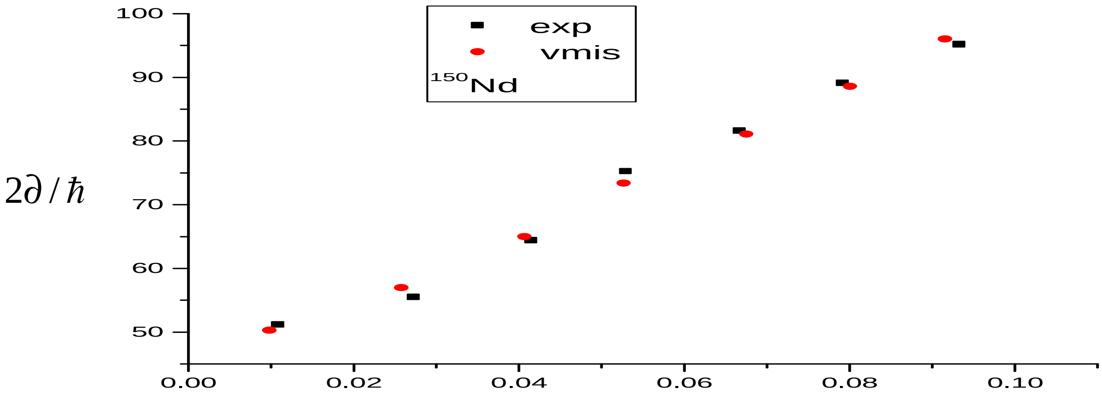

Figure 15: $(\hbar\omega)^2$